

LABORATORY NO. 08:  
INVERTED PENDULUM

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BYU-Idaho CompE 470

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## I. OBJECTIVES

The objective for this laboratory is to design, implement, and document both Full State and Sliding Mode Control for the Inverted Pendulum.

## II. PRELIMINARY

Obtain a copy of the Quanser Inverted Pendulum data sheet in order to get the approximated state equation and gain values that will give a good starting value for the lab. Using these values, design a full state feedback controller which will keep the pendulum vertical. When finished with the full state controller, design a sliding mode controller to do the same thing.

### A. Formulas and Overall Descriptions Used

Most of this laboratory's calculations were completed and compiled by the folks at Quanser (the manufacturer of the inverted pendulum) and will give the lab a good starting place. Below are the state equation and gain values used initially in the lab:

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 81.7 & 0 & 0 & -13.9 \\ 0 & 0 & 0 & 1 \\ 39.7 & 0 & 0 & -14.4 \end{bmatrix} \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 24.5 \\ 0 \\ 25.4 \end{bmatrix} V$$

$$K = [21 \quad 2.8 \quad -2.2 \quad -2.0]$$

Other values, such as the  $\frac{\text{Volts}}{\text{Degree}}$  and  $\frac{\text{Degrees}}{\text{Volt}}$  were obtained by first determining the max angle of the pendulum on both extreme sides (see Figure 1 below).

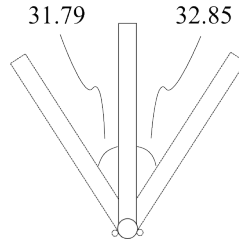


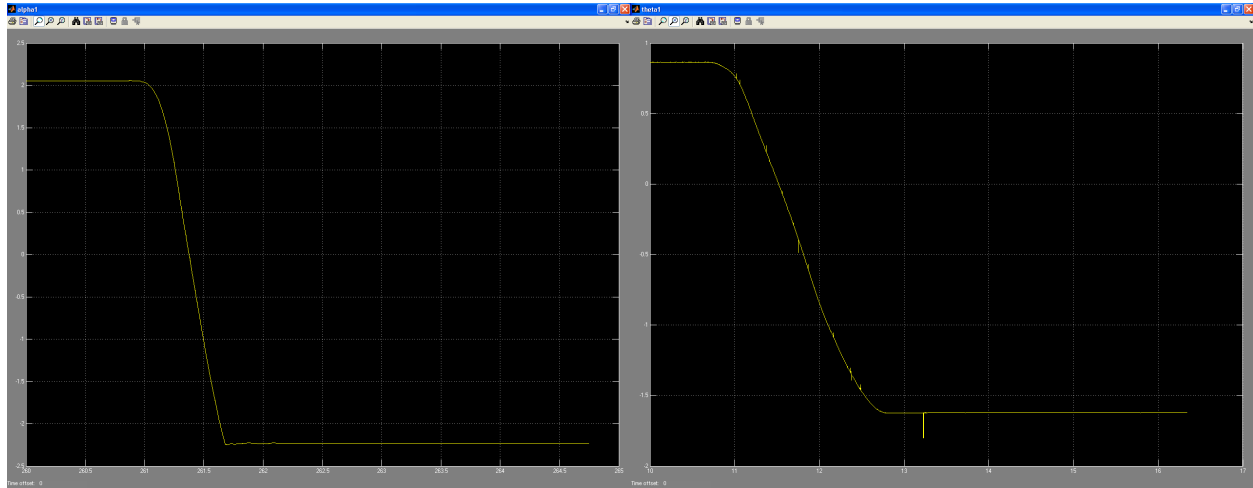
FIG. 1: Extreme angles for the inverted pendulum and the corresponding angles.

Using the max angles from above, these values were determined:

$$\alpha = 0.062 \frac{\text{Volts}}{\text{Degree}}$$

$$\alpha = 15.105 \frac{\text{Degrees}}{\text{Volt}}$$

I would also like to add that in order to calibrate  $\alpha$  to get a perfect vertical = 0, a value of 0.09 needed to be added. The same applies to  $\theta$  where 0.322 needs to be added. The graphs which helped determine this are in Figures 2a and 2b below.



(a)  $\alpha_{max} = 2.052$  and  $\alpha_{min} = -2.23$

(b)  $\theta_{max} = 0.896$  and  $\theta_{min} = -1.539$

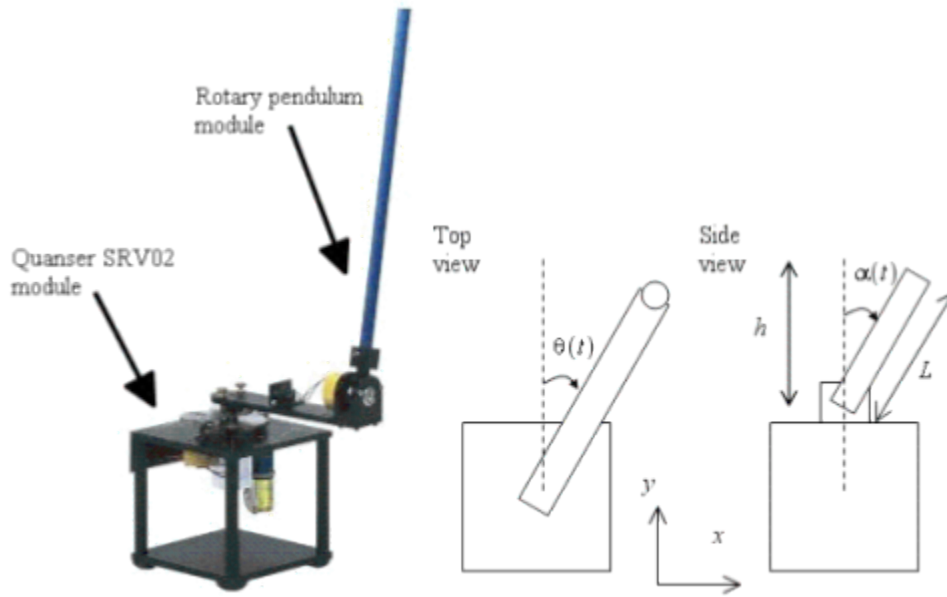
FIG. 2: Extreme voltages for both  $\alpha$  and  $\theta$ .

As a guide to how the system works, here is a quick outline highlighting the important roles of  $\alpha$  and  $\theta$ :

- $\alpha$  controls the swing of  $\theta$
- Adding too much  $\theta$  caused the system to become unstable
- The more  $\dot{\alpha}$  was increased the more it would want to compensate for its offset and the same is for  $\dot{\theta}$

## B. Overall Diagrams

This section will consist of the diagrams and schematics used for this laboratory.



(a) Quanser single inverted rotary pendulum system

(b) Schematic representation of the system

FIG. 3: Description of Single Inverted Rotary Pendulum (SIRP) System

### III. PROCEDURES

#### A. Equipment

- Quanser Inverted Pendulum apparatus (pre-assembled)
- MatLab 2007b Software
- National Instruments hardware to connect to MatLab 2007b

#### B. Schematics

There were no circuit schematics used in this laboratory. However, there were block diagrams used. Please refer to Figure ?? for an overall view of the system within MatLab.

#### C. Circuit Results

There are no circuit results to be presented.

#### D. Explanation and Data

Most of the data, aside from those values given from the Quanser datasheet, were guess and check. In other words, we used what was given to us (see preliminary) and then we tweaked it to our liking. The final values that we used were:

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 81.7 & 0 & 0 & -13.9 \\ 0 & 0 & 0 & 1 \\ 39.7 & 0 & 0 & -14.4 \end{bmatrix} \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 24.5 \\ 0 \\ 25.4 \end{bmatrix} V$$

$$K_{fullstate} = [75 \quad 3.6 \quad -3.0 \quad -2.65]$$

$$K_{slidingmode} = [-15 \quad -0.01 \quad 1.5 * 2 \quad 0.0005]$$

#### E. Simulations

Below are screengrabs from the simulations of this laboratory. Since most of the simulation information is in the section above (Explanation and Data), I will not be expanding upon these graphs.

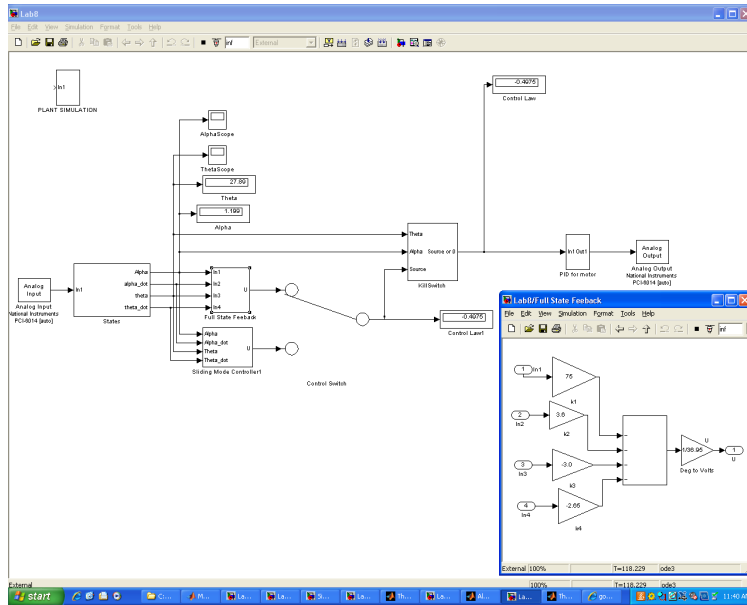


FIG. 4: Full State Feedback Controller

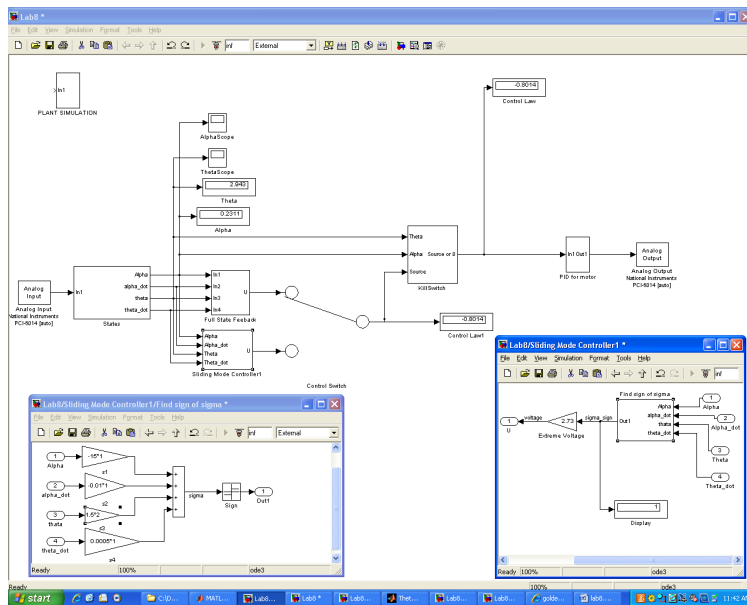


FIG. 5: Sliding Mode Controller

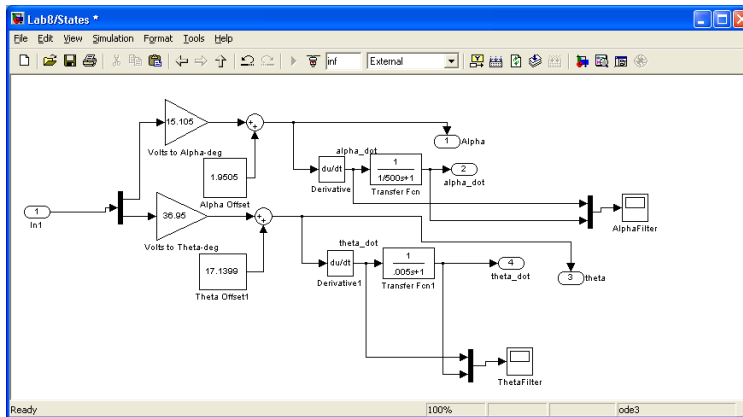


FIG. 6: State Equation Subsystem

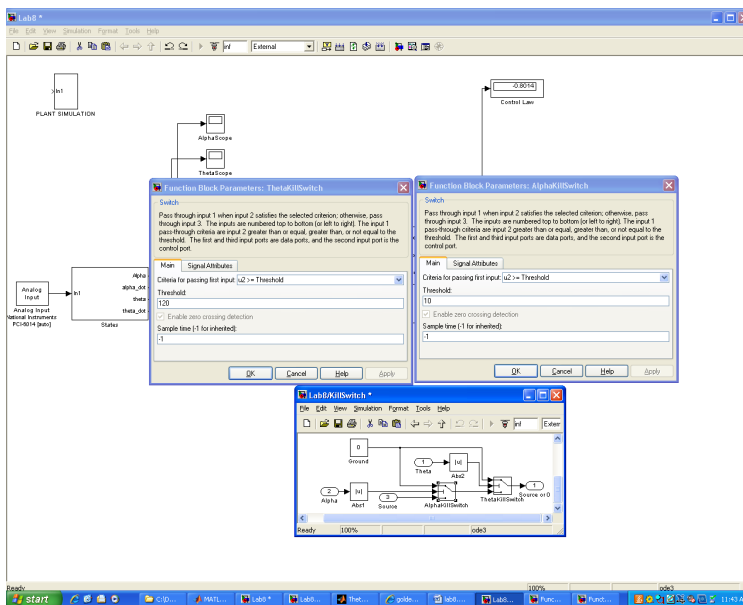


FIG. 7: The “Kill Switch” of the system and thresholds

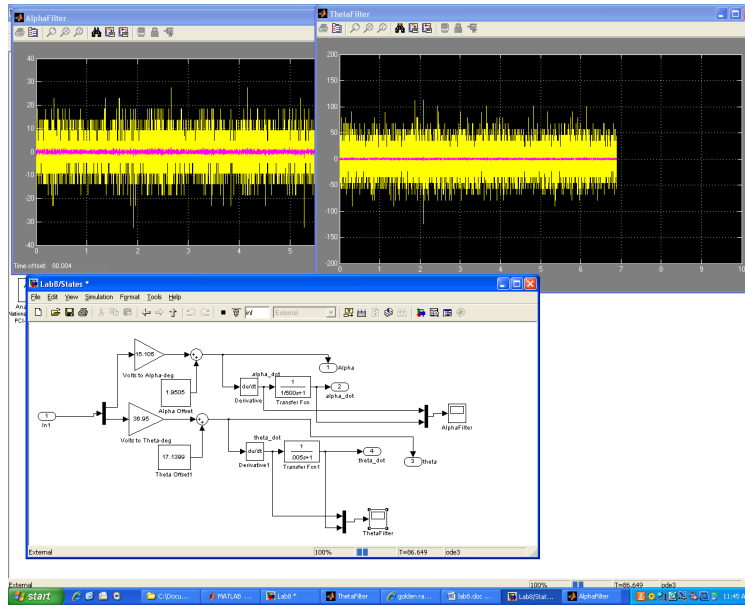


FIG. 8: The filters of the system.

#### IV. CONCLUSION

This laboratory taught students how to design, implement, and document both Full State and Sliding Mode Control for the Inverted Pendulum.

As a group, we struggled to grasp the concepts at first, but we slowly inched forward, making changes to the system little by little. We started off with making sure we connected the inverted pendulum correctly to MATLAB and then moved onto making sure our angles and zeros (verticals) were correctly coordinated within MATLAB. From there, we started to design the full state and then the sliding mode controller. Our full state controller was incredibly smooth and stable. Sometimes, it didn't even look like the machine was working other than noticing that the pendulum was inverted. The sliding mode controller worked well, but it chattered because of the motor going from positive to negative so quickly.

Personally, I struggled understanding the lab as a whole. I could understand the concepts of full state and sliding mode, but I still couldn't exactly tell you how to implement them in a lab like this. Not to search too hard for excuses, but being in a lab group of 4, working at a desk which holds only 2 can be difficult to become involved in. There's just simply not enough room. In the future, I would recommend that this lab be done in smaller groups, and spread the due dates apart possibly. The most important part is that I learn to work in a group this size and be able to keep up and ask questions when I need to.

Overall, this lab was very interesting. I was thrilled to see the pendulum being balanced perfectly by the controller we designed in MATLAB even when I couldn't do it by hand. This lab probably taught me the most about what a little feedback control can really accomplish. As the final lab of the semester, I regret that I couldn't see feedback control used in several different situations rather than the two we studied here.