State Reduction Handout

- Three Major Methods
  - Row Matching
  - Implication Chart Method
  - Successive Partitioning Method

- Row Matching
  - Recipe
    1. Start with the state transition table
    2. Identify states with same output behavior
    3. If such states transition to the same next state, they are equivalent
    4. Combine into a single new renamed state
    5. Repeat until no new states are combined

- Implication Chart Method
  - Entry Xij -- Row is Si, Column is Sj
  - Recipe
    6. Construct implication chart, one square for each combination of states taken two at a time
    7. Square labeled Si, Sj, if outputs differ than square gets "X". Otherwise write down implied state pairs for all input combinations
    8. Advance through chart top-to-bottom and left-to-right. If square Si, Sj contains next state pair Sm, Sn and that pair labels a square already labeled "X", then Si, Sj is labeled "X".
    9. Continue executing Step 3 until no new squares are marked with "X".
    10. For each remaining unmarked square Si, Sj, then Si and Sj are equivalent.

- Successive Partitioning Method:
  - Partitioning provides a straightforward procedure for determining equivalency for any amount of complexity. Successive partitioning steps produce smaller partitions. If the next step does not yield any smaller partitions no further steps will yield any smaller partitions and, hence, the partitioning process is then complete. All states that are in the same partition after k steps are k equivalent. All states that are in the same partition when no further partitioning can be accomplished are equivalent. States that are not in the same final partition are not equivalent.

Row Matching Method

1.) Initial State Transition Table

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S7	S8	0	0
01	S4	S9	S10	0	0
10	S5	S11	S12	0	0
11	S6	S13	S14	0	0
000	S7	S0	S0	0	0
001	S8	S0	S0	0	0
010	S9	S0	S0	0	0
011	S10	S0	S0	1	0
100	S11	S0	S0	0	0
101	S12	S0	S0	1	0
110	S13	S0	S0	0	0
111	S14	S0	S0	0	0

2.) Identify states with same output behavior.
3.) If such states transition to the same next state, they are equivalent.

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S7	S8	0	0
01	S4	S9	S10	0	0
10	S5	S11	S12	0	0
11	S6	S13	S14	0	0
000	S7	S0	S0	0	0
001	S8	S0	S0	0	0
010	S9	S0	S0	0	0
011	S10	S0	S0	1	0
100	S11	S0	S0	0	0
101	S12	S0	S0	1	0
110	S13	S0	S0	0	0
111	S14	S0	S0	0	0

4.) Combine into a single new renamed state.

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S7	S8	0	0
01	S4	S9	S10	0	0
10	S5	S11	S10'	0	0
11	S6	S13	S14	0	0
000	S7	S0	S0	0	0
001	S8	S0	S0	0	0
010	S9	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0
100	S11	S0	S0	0	0
110	S13	S0	S0	0	0
111	S14	S0	S0	0	0

Removed S12, replace all S12 states with S10'

5.) Repeat until no new states are combined....

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S7	S8	0	0
01	S4	S9	S10'	0	0
10	S5	S11	S12	0	0
11	S6	S13	S14	0	0
000	S7	S0	S0	0	0
001	S8	S0	S0	0	0
010	S9	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0
100	S11	S0	S0	0	0
110	S13	S0	S0	0	0
111	S14	S0	S0	0	0

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S7'	S7'	0	0
01	S4	S7'	S10	0	0
10	S5	S7'	S7'	0	0
11	S6	S7'	S7'	0	0
Not (011 or 101)	S7'	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0

Removed S8-S9 and S11-S14, replace all with S7'

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S7'	S7'	0	0
01	S4	S7'	S10	0	0
10	S5	S7'	S7'	0	0
11	S6	S7'	S7'	0	0
Not (011 or 101)	S7'	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3'	S4	0	0
1	S2	S5	S6	0	0
00	S3'	S7'	S7'	0	0
01	S4	S7'	S10	0	0
10	S5	S7'	S7'	0	0
11	S6	S7'	S7'	0	0
Not (011 or 101)	S7'	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0

Removed S6, replace with S3'

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3'	S4	0	0
1	S2	S5	S3'	0	0
00 or 11	S3'	S7'	S7'	0	0
01	S4	S7'	S10	0	0
10	S5	S7'	S7'	0	0
Not (011 or 101)	S7'	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3'	S4'	0	0
1	S2	S4'	S3'	0	0
00 or 11	S3'	S7'	S7'	0	0
01 or 10	S4'	S7'	S10	0	0
Not (011 or 101)	S7'	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0

Removed S5, replace all with S4'

Final State Transition Table

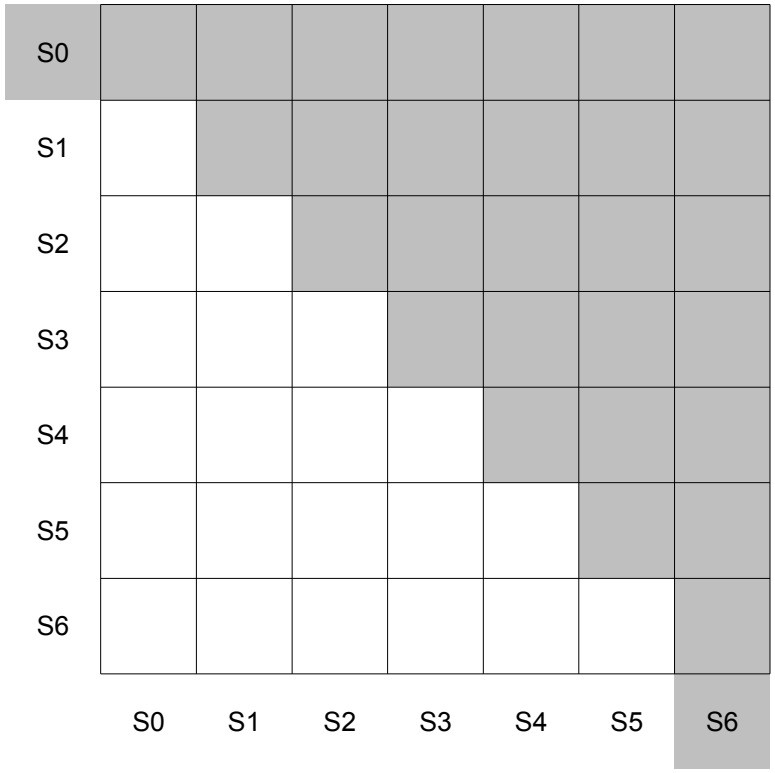
Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3'	S4'	0	0
1	S2	S4'	S3'	0	0
00 or 11	S3'	S7'	S7'	0	0
01 or 10	S4'	S7'	S10	0	0
Not (011 or 101)	S7'	S0	S0	0	0
011 OR 101	S10'	S0	S0	1	0

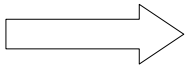
Implication Chart Method

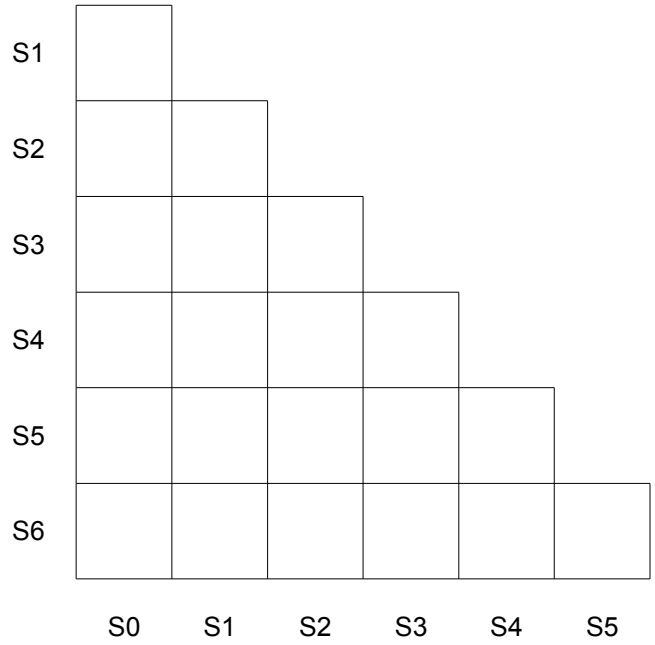
1.) Single Input Example

Initial State Transition Table

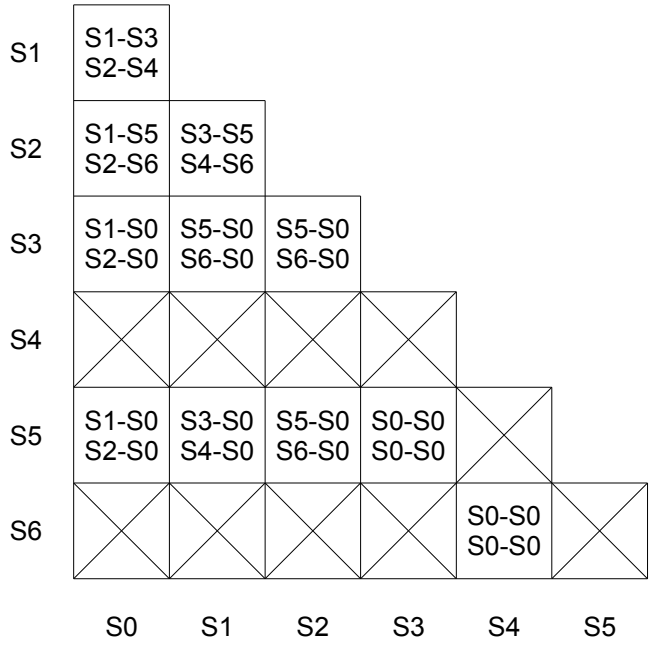
Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S0	S0	0	0
01	S4	S0	S0	1	0
10	S5	S0	S0	0	0
11	S6	S0	S0	1	0

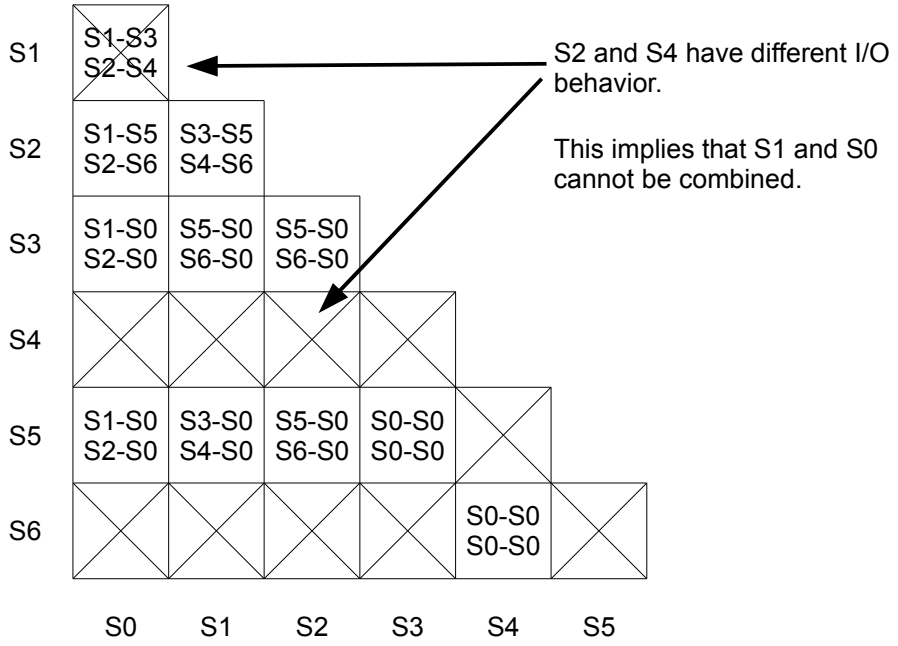


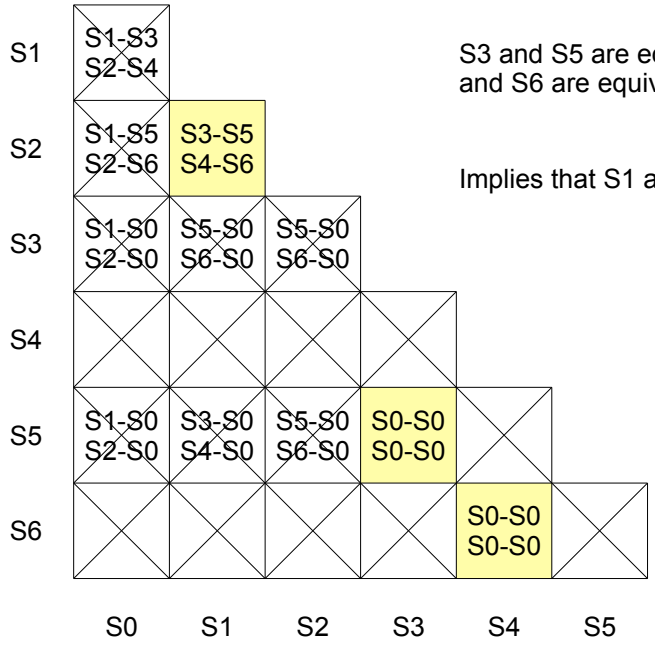




Implication







S4

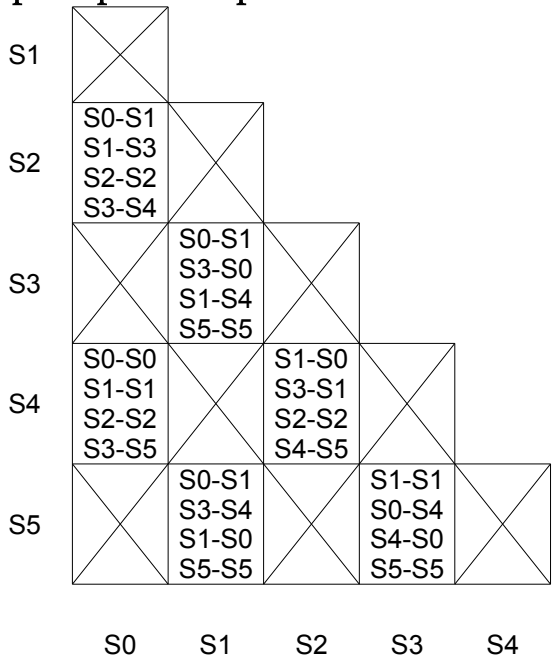
State Transition Table Simplification

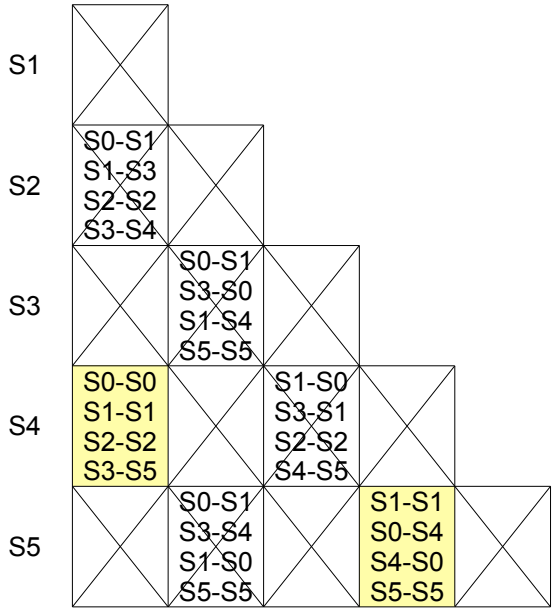
Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1	S2	0	0
0	S1	S3	S4	0	0
1	S2	S5	S6	0	0
00	S3	S0	S0	0	0
01	S4	S0	S0	1	0
10	S5	S0	S0	0	0
11	S6	S0	S0	1	0

Final State Transition Table

Input Sequence	Present State	Next State X=0	Next State X=1	Output X=0	Output X=1
Reset	S0	S1'	S1'	0	0
0 or 1	S1'	S3'	S4'	0	0
00 or 10	S3'	S0	S0	0	0
01 or 11	S4'	S0	S0	1	0

2.) Multiple Input Example





Initial State Transition Table

Present State	Next State 00	01	10	11	Output
S0	S0	S1	S2	S3	1
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

State Transition Table Simplification

Present State	Next State 00	01	10	11	Output
S0	S0	S1	S2	S3	1
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

Final State Transition Table

Present State	Next State 00	01	10	11	Output
S0'	S0'	S1	S2	S3'	1
S1	S0	S3'	S1	S3'	0
S2	S1	S3'	S2	S0'	1
S3'	S1	S0'	S0'	S3'	0

Successive Partitioning Method

Initial Transition Table:

	NS				Z			
PS	**00**	**01**	**10**	**11**	**00**	**01**	**10**	**11**
A	A	H	G	G	1	0	1	0
B	C	H	G	F	1	0	1	0
C	A	H	G	A	1	0	1	0
D	D	D	F	F	0	0	0	0
E	H	H	F	E	1	0	0	0
F	H	H	E	F	0	0	0	0
G	A	H	A	A	1	0	1	0
H	H	G	H	G	1	0	1	0

Initial Machine Table:

	00	01	10	11
A	A/1	H/0	G/1	G/0
B	C/1	H/0	G/1	F/0
C	A/1	H/0	G/1	A/0
D	D/0	D/0	F/0	F/0
E	H/1	H/0	F/0	E/0
F	H/0	H/0	E/0	F/0
G	A/1	H/0	A/1	A/0
H	H/1	G/0	H/1	G/0

Final Reduced Transition Table:

	NS				Z			
PS	**00**	**01**	**10**	**11**	**00**	**01**	**10**	**11**
A'	A'	A'	A'	A'	1	0	1	0
B'	A'	A'	A'	E'	1	0	1	0
C'	C'	C'	E'	E'	0	0	0	0
D'	A'	A'	E'	D'	1	0	0	0
E'	A'	A'	D'	E'	0	0	0	0

Partitioning Method:

Step 1:

P0	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
x1,x2	OUTPUTS							
00	1	1	1	0	1	0	1	1
01	0	0	0	0	0	0	0	0
10	1	1	1	0	0	0	1	1
11	0	0	0	0	0	0	0	0
P1	**A**	**B**	**C**	**G**	**H**	**D**	**F**	**E**

Step 2:

P1	**A**	**B**	**C**	**G**	**H**	**D**	**F**	**E**
x1,x2								
00	A	C	A	A	H	D	H	
01	H	H	H	H	G	D	H	
10	G	G	G	A	H	F	E	
11	G	F	A	A	G	F	F	
P2	**A**	**C**	**G**	**H**	**B**	**D**	**F**	**E**

Final Step 3:

P1	**A**	**C**	**G**	**H**	**B**	**D**	**E**	**F**
x1,x2								
00	A	A	A	H				
01	H	H	H	G				
10	G	G	A	H				
11	G	A	A	G				
P3	**A**	**C**	**G**	**H**	**B**	**D**	**E**	**F**

Composite Partition Table:

P0	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
x1,x2	OUTPUTS							
00	1	1	1	0	1	0	1	1
01	0	0	0	0	0	0	0	0
10	1	1	1	0	0	0	1	1
11	0	1	1	0	0	0	1	1
P1	**A**	**B**	**C**	**G**	**H**	**D**	**F**	**E**
00	A	C	A	A	H	D	H	
01	H	H	H	H	G	D	H	
10	G	G	G	A	H	F	E	
11	G	H	A	A	G	F	F	
P2	**A**	**C**	**G**	**H**	**B**	**D**	**E**	**F**
00	A	A	A	H				
01	H	H	H	G				
10	G	G	A	H				
11	G	A	A	G				
P2	**A**	**C**	**G**	**H**	**B**	**D**	**E**	**F**

Equivalences:

A' = A = C = G = H F
B' = B
C' = D
D' = E
E' = F

Final Reduced Machine Table:

	X1,X2			
Q	00	01	10	11
A'	A'/1	A'/0	A'/1	A'/0
B'	A'/1	A'/0	A'/1	E'/0
C'	C'/0	C'/0	E'/0	E'/0
D'	A'/1	A'/0	E'/0	D'/0
E'	A'/0	A'/0	D'/0	E'/0

State Reduction Problems

1.) Reduce the following state table to the minimum number of states using successive partitioning method.

	NS		Z	
PS	X=0	X=1	X=0	X=1
A	A	E	1	0
B	C	F	0	0
C	B	H	0	0
D	E	F	0	0
E	D	A	0	1
F	B	F	1	1
G	D	H	0	1
H	H	G	1	0

2.) Reduce the following state table to the minimum number of states using implication chart method.

	NS		
PS	X=0	X=1	Z
A	E	E	1
B	C	E	1
C	I	H	0
D	H	A	1
E	I	F	0
F	E	G	0
G	H	B	1
H	C	D	0
I	F	B	1

3.) Repeat problem 1 using the implication chart method.

4.) Repeat problem 2 using the successive partitioning method.